Projection-Based Model Order Reduction as Bayesian Conditioning

Anne Poot[1*], Iuri Rocha[1], Pierre Kerfriden[2], Frans van der Meer[1]

[1]Faculty of Civil Engineering and Geosciences
Delft University of Technology
Stevinweg 1, 2628 CN Delft, The Netherlands
{a.poot-1,i.rocha,f.p.vandermeer}@tudelft.nl

[2]Centres des Matériau
Mines Paris—PSL
63-65 Rue Henri Auguste Desbruères, 91100 Évry, France
pierre.kerfriden@minesparis.psl.eu

ABSTRACT

A common challenge in methods for uncertainty quantification (e.g. uncertainty propagation, inverse modeling and data assimilation) is that they typically require many model evaluations in order to propagate the uncertainty in the inputs to the quantity of interest. Especially when the model evaluation requires solving a partial differential equation numerically, computational expenses can skyrocket. To alleviate these computational costs, projection-based model order reduction techniques (proper orthogonal decomposition, balanced truncation, reduced basis methods, etc.) are often applied, which aim to minimize the model complexity. However, these reduced-order models unavoidably introduce an approximation error, presenting the modeler with a difficult trade-off between accuracy and computational cost. Often, it is unclear what is the effect of neglecting higher-order modes on the distributions that are obtained for the quantity of interest.

In this work, we present a Bayesian formulation of projection-based reduced order models. The full-order model is endowed with a carefully chosen Gaussian prior distribution, for which each basis function of the reduced-order model functions as an observation. Performing the Bayesian conditioning yields a posterior distribution whose mean recovers the classic reduced-order model solution and whose posterior covariance can be related directly to the reduced-order model error. By modeling this error probabilistically, it can be taken into account consistently by propagating it to the quantity of interest. We present a theoretical description of the method, along with an empirical study applying our Bayesian reduced-order model to a Bayesian inverse problem.